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THE EARTH AS A GIGANTIC, RAPIDLY ROTATING GYROSCOPE†

D. R. MERKIN

St Petersburg

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A method of calculating the earth's precession, known from observations, with a period of 26 000 years, is proposed. It is suggested that this precession can be calculated using gyroscope precession theory, assuming the earth itself to be a rapidly rotating gyroscope. The earth's precession is assumed to be due to the action of the moments of the attractive forces of the sun and the moon.

Even Hipparch, comparing his observations with those of his predecessors, found that the earth's axis makes one revolution about a stationary axis of precession in approximately 26 000 years, the precession of the earth's axis occurring in the opposite direction to the apparent motion of the sun. Comparing the angular velocity ω of daily rotation of the earth with the angular velocity ω_π of the precession of its axis, we find $\omega/\omega_\pi \sim 9.5 \times 10^6$. This gives grounds for considering the earth to be a gigantic, rapidly rotating gyroscope to which it is possible to apply elementary theory and simplify significantly the theoretical substantiation of the precession of the earth's axis.

We will construct a right-handed orthogonal coordinates system of $Oxyz$ in which the origin O is located at the earth's centre, the z axis is directed along the earth's axis of proper rotation towards the North Pole and the x axis is directed along nodal line $O\Upsilon$; then the y axis is positioned in the earth's equatorial plane Q–Q'. Let S be the position of the sun on the ecliptic E–E'. The arc $\Upsilon S = \lambda$ defines the longitude of the sun, which is always measured from the nodal line in the direction of the apparent motion of the sun. The position of the moving trihedron in relation to the stationary axes $Q\xi\eta\zeta$ (the $O\xi$ and $O\eta$ axes are positioned in the ecliptic plane) is defined by the angle of precession ψ and the angle ϑ between the ζ and z axes (see Fig. 1); the $O\xi$ axis is chosen arbitrarily.

Under the action of the attractive forces of the sun and moon, the angle ϑ performs small nutational oscillations. These oscillations under the action of the sun were first mentioned by Newton [1] (without derivation), noting that they occur twice a year and are barely perceptible (their amplitude is 0.50″). On this basis, we will ignore, as usual, the nutational oscillations and will consider the angle ϑ to be constant, determined, according to observations, by the equality

$$\vartheta = 23°27' \tag{1}$$

The problem consists of determining, using the law of precession of the axis of a gyroscope, the variation of the angle ψ due to the action of the attractive forces of the sun and the moon.

We will give well-known formulae for the moments of the gravitational forces of the sun with which they act on the earth ($A = B$, C are the moments of inertia of the earth)

$$\begin{aligned} M_x &= 3\frac{fM}{r^3}(C-A)\sin\vartheta\cos\vartheta\sin^2\lambda = \varkappa\sin\vartheta\cos\vartheta(1-\cos2\lambda) \\ M_y &= 3\frac{fM}{r^3}(A-C)\sin\vartheta\sin\lambda\cos\lambda = -\varkappa\sin\vartheta\sin2\lambda \\ M_z &= 0 \end{aligned} \tag{2}$$

where f is the gravitational constant, M is the mass of the sun, r is the average distance from the sun to the earth and the constant $\varkappa$ is defined by the equality

$$\varkappa = \frac{3}{2}\frac{fM}{r^3}(C-A) = \frac{3}{2}\omega_0^2(C-A) \tag{3}$$

where $\omega_0^2 = fM/r^3$ is the square of the angular velocity of rotation of the sun–earth line (strictly speaking, $\omega_0^2 = (fM/r^3)(1 + m/M)$, but $m/M \approx 3 \times 10^{-6}$).

Evaluating the integrals with respect to λ of M_x and M_y, we find that the average annual value of the moments of the gravitational forces of the sun is directed along the nodal line $O\Upsilon$ and is defined by the equality

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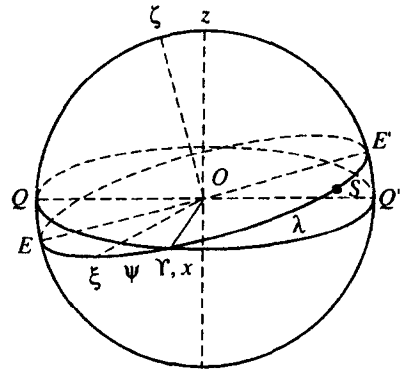

Fig. 1

$$\langle M_S \rangle = \frac{3}{2}\frac{fM}{r^3}(C-A)\sin\vartheta\cos\vartheta \tag{4}$$

We will now calculate the annual average value of the moments of the gravitational forces of the moon, considering its orbit in relation to the earth to be circular. Taking into account that the masses of the earth and the moon are of the same order, we determine the angular velocity ω_1 of the moon relative to the earth from the equality

$$\omega_1^2 = \frac{fm_1}{r_1^3}\left(1+\frac{m}{m_1}\right) \tag{5}$$

where m is the mass of the earth, m_1 is the mass of the moon and r_1 is the average distance from the earth to the moon.

The average annual value of the moments of the gravitational forces of the moon can be obtained from Eq. (4) if the quantity fM/r^3 is replaced by fm_1/r_1^3, determined from equality (5):

$$\langle M_M \rangle = \frac{3}{2}\frac{\omega_1^2}{(1+m/m_1)}(C-A)\sin\vartheta\cos\vartheta \tag{6}$$

We will now use the law of precession of the gyroscope axis

$$d\mathbf{K}/dt = \mathbf{u} \tag{7}$$

where $\mathbf{K} = C\omega$ and $\mathbf{u}$ is the linear velocity of the end of the vector $\mathbf{K}$, equal in modulus and collinear to the moment of the external forces. The angle between the vector ω and the direction of the vector of the angular velocity of precession is equal to $\pi - \vartheta$. Consequently, the angular velocity of precession of the earth's axis will be equal to

$$\dot{\psi} = \frac{\langle M_S \rangle + \langle M_M \rangle}{C\omega\sin(\pi-\vartheta)}$$

Using equalities (4) and (6) and $fM/r^3 = \omega_0^2$, we obtain

$$\dot{\psi} = -\frac{3}{2}\left[\frac{\omega_0^2}{\omega} + \frac{\omega_1^2}{(1+m/m_1)\omega}\right]\frac{C-A}{C}\cos\vartheta$$

Integrating this equality with respect to t from 0 to $2\pi/\omega_0$, we find the change in the angle of precession per year

$$\psi = -3\pi\left[\frac{\omega_0}{\omega} + \frac{1}{(1+m/m_1)}\frac{\omega_1}{\omega}\frac{\omega_1}{\omega}\right]\frac{C-A}{C}\cos\vartheta \tag{8}$$

For the sun–earth–moon system we have

$$\frac{\omega_0}{\omega} = \frac{1}{365\cdot 25}, \quad \frac{\omega_1}{\omega} = \frac{1}{27\cdot 32}, \quad \frac{m}{m_1} = 81{,}5 \tag{9}$$

Furthermore, for the earth it has been established that

$$(C-A)/C = 0.00325 \tag{10}$$

Using Eqs (1), (9) and (10) and dividing Eq. (8) into two parts, we find the annual change in the angle ψ that occurs from the attractive forces of the sun and the moon

$$\psi_S = -15.87, \quad \psi_M = -34.38$$

In the simplifying assumptions made, the total precession per year is equal to 50.25″, which is very similar to the directly observed magnitude of the annual precession of 50.236″. (Hipparch made an error of less than 1%; in fact, the time of a complete revolution of the earth's axis about the ecliptic axis is (in years) $360°/\psi$ = 360 60 60/50.236 = 25 798.)

The method used here is much simpler and shorter than the use of Euler's equations, and each action is physical clear.

REFERENCE

1. Newton, I., Mathematical principles of natural philosophy. In *Collected Papers of Academician A. N. Krylov*, Vol. 7. Izd. Akad. Nauk SSSR, Moscow, 1936.

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